Lecture Notes
Graph Theory
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Based upon "Graph Theory", Bondy and Murty, Springer, 2008

This text is an update and extension of a classic but out-of-print text which can be downloaded at *http://www.ecp6.jussieu.fr/pageperso/bondy/books/gtwa/gtwa.html* .

Chapter One

Graph $(V(G),E(G),\psi_G)$, where
V(G) = set of vertices
E(G) = set of edges
ψ_G = function relating edges to endpoint vertices

$n = |V(G)|$
$\varepsilon = |E(G)|$

Adjacent vertices, neighbors, neighborhood $N_G(u)$

Loop, parallel edges

Finite Graphs have finite n and ε.

Null graph has no vertices
Trivial graph has only one vertex

Simple graph has no loops or parallel edges

Complete graph, empty graph, bipartite graph

Path, Cycle, length, even, odd

Connected graph

Planar graph, planar embedding

Incidence Matrix = M
Adjacency Matrix = A, and adjacency list (does not store the 0's)
Bipartite Adjacency Matrix

d(v)
$\delta(G) = \min(d(v))$
$\Delta(G) = \max(d(v))$
d(G) = average of d(v)

Theorem 1.1 – pg 7
Pf: Consider M by rows and then by columns

Corollary 1.2 – pg 7
Pf: $2\varepsilon = \sum$even d(v) + $\sum$ odd d(v) = even + $\sum$ odd d(v) implies last term is even.

k-regular if d(v) = k, for all v

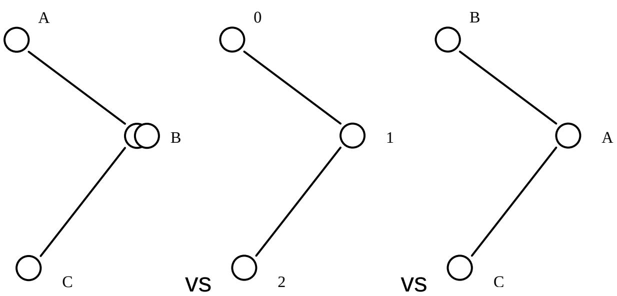
cubic graph for k = 3

Proposition 1.3 - pg 8

Homework – pg 9
1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 14, 16, 17

Identical graphs

Ex:



Isomorphic G≅H

Note: For isomorphic graphs, φ is completely determined by θ if the graphs are simple. Indeed, $\varphi(e)=\theta(u)\theta(v)$, whenever $\psi(e) = uv$.

K_n, $K_{m,n}$, P_n, C_n

Testing graph for isomorphisms
1. Same number of vertices?
2. Same number of edges?
3. Same number of vertices of similar degree?
4. Check mappings between vertices of similar degree.
5. For a simple graph, given a θ, check $uv \in E(G) \Leftrightarrow \theta(u)\theta(v) \in E(H)$. $\binom{n}{2}$ pairs.

Note: There are n! possible bijections between two graphs with n vertices each.

Automorphism. For simple graphs, an automorphism is an permutation of the vertices of the graph. Paired vertices are called similar.

Labeled graphs

Result: There are $2^{\binom{n}{2}}$ labeled simple graph on a graph with n vertices.

The set of all graph on a given vertex set is a group called G_n.

Result: The number of unlabelled simple graph on n vertices is at least $\left\lceil \frac{2^{\binom{n}{2}}}{n!} \right\rceil$.

Homework – page 17
 # 1, 2, 5, 6, 9, 14, 15

Polyhedral graphs – pg 21

Platonic graphs
Hypercubes

Homework – page 24
 # 2

Disjoint graph
Union of graphs, $G \cup H$ or G+H
Components – pg 29. c(G) = number of components
Intersection of graphs , $G \cap H$
Cartesian Product, $G \square H$

$P_m \square P_n$ = mxn grid
$C_m \square K_2$ = m-prism

Homework – page 31
#1, 3, 4

Directed graph “digraph”
Tournament – an orientation of K_n.
Converse – pg 33
Principle of Directional Duality – pg 33

Homework – page 34
 # 1, 2, 3, 5, 6, 7, 8, 11

Infinite graph – pg 36

Homework – page 36
#1, 3